

演習：不定積分の計算（基礎）【微分積分】

1.

$$(1) \int dx x^\alpha \quad (\alpha \neq -1)$$

$$(5) \int dx \cos x$$

$$(9) \int dx e^x$$

$$(2) \int \frac{dx}{x}$$

$$(6) \int dx \sin x$$

$$(10) \int dx \ln x$$

$$(3) \int dx \sqrt{x}$$

$$(7) \int \frac{dx}{\cos^2 x}$$

$$(11) \int dx a^x$$

$$(4) \int \frac{dx}{\sqrt{x}}$$

$$(8) \int \frac{dx}{\sin^2 x}$$

$$(12) \int dx \log_a x$$

2.

$$(1) \int dx x(x^2 + 1)^3$$

$$(5) \int dx \sin^2 x \cos x$$

$$(9) \int dx \frac{e^x}{e^x + 1}$$

$$(2) \int dx \frac{x^2}{x^3 + 1}$$

$$(6) \int dx \sin x \cos x$$

$$(10) \int dx \frac{\ln x}{x}$$

$$(3) \int dx \frac{x}{\sqrt{x^2 + 1}}$$

$$(7) \int dx \tan x$$

$$(11) \int \frac{dx}{x \ln x}$$

$$(4) \int dx x \cos(x^2)$$

$$(8) \int dx x e^{x^2}$$

$$(12) \int dx e^{\sin x} \cos x$$

3.

$$(1) \int dx x e^x$$

$$(7) \int dx x \ln x$$

$$(2) \int dx x \cos x$$

$$(8) \int dx x^2 \ln x$$

$$(3) \int dx x \sin x$$

$$(9) \int dx (\ln x)^2$$

$$(4) \int dx x^2 e^x$$

$$(10) \int dx x (\ln x)^2$$

$$(5) \int dx x^2 \cos x$$

$$(11) \int dx e^x \cos x$$

$$(6) \int dx x^2 \sin x$$

$$(12) \int dx e^x \sin x$$

4.

$$(1) \int dx x(x-1)^5$$

$$(7) \int dx \sin^2 x$$

$$(2) \int dx \frac{x}{x+1}$$

$$(8) \int dx \cos^3 x$$

$$(3) \int \frac{dx}{x^2 - 1}$$

$$(9) \int dx \sin(2x) \sin(3x)$$

$$(4) \int dx \frac{x}{\sqrt{x+1}}$$

$$(10) \int dx \sin(x) \cos(2x)$$

$$(5) \int dx x \sqrt{x-1}$$

$$(11) \int \frac{dx}{1 + \sin x}$$

$$(6) \int \frac{dx}{\sqrt{x^2 - 1}}$$

$$(12) \int \frac{dx}{\sin x}$$

演習：不定積分の計算（基礎）【微分積分】解答

1.

$$\begin{array}{lll}
 (1) \int dx x^\alpha = \frac{x^{\alpha+1}}{\alpha+1} \quad (\alpha \neq -1) & (5) \int dx \cos x = \sin x & (9) \int dx e^x = e^x \\
 (2) \int \frac{dx}{x} = \ln|x| & (6) \int dx \sin x = -\cos x & (10) \int dx \ln x = x(\ln x - 1) \\
 (3) \int dx \sqrt{x} = \frac{2x\sqrt{x}}{3} & (7) \int \frac{dx}{\cos^2 x} = \tan x & (11) \int dx a^x = \frac{a^x}{\ln a} \\
 (4) \int \frac{dx}{\sqrt{x}} = 2\sqrt{x} & (8) \int \frac{dx}{\sin^2 x} = -\frac{1}{\tan x} & (12) \int dx \log_a x = \frac{x(\ln x - 1)}{\ln a}
 \end{array}$$

2.

$$\begin{array}{lll}
 (1) \int dx x(x^2+1)^3 = \frac{1}{8}(x^2+1)^4 & (5) \int dx \sin^2 x \cos x = \frac{1}{3} \sin^3 x & (9) \int dx \frac{e^x}{e^x+1} = \ln(e^x+1) \\
 (2) \int dx \frac{x^2}{x^3+1} = \frac{1}{3} \ln|x^3+1| & (6) \int dx \sin x \cos x = \frac{1}{2} \sin^2 x & (10) \int dx \frac{\ln x}{x} = \frac{(\ln x)^2}{2} \\
 (3) \int dx \frac{x}{\sqrt{x^2+1}} = \sqrt{x^2+1} & (7) \int dx \tan x = -\ln|\cos x| & (11) \int \frac{dx}{x \ln x} = \ln|\ln x| \\
 (4) \int dx x \cos(x^2) = \frac{1}{2} \sin(x^2) & (8) \int dx x e^{x^2} = \frac{e^{x^2}}{2} & (12) \int dx e^{\sin x} \cos x = e^{\sin x}
 \end{array}$$

3.

$$\begin{array}{ll}
 (1) \int dx x e^x = (x-1)e^x & (7) \int dx x \ln x = \frac{x^2}{4}(2 \ln x - 1) \\
 (2) \int dx x \cos x = \cos x + x \sin x & (8) \int dx x^2 \ln x = \frac{x^3}{9}(3 \ln x - 1) \\
 (3) \int dx x \sin x = \sin x - x \cos x & (9) \int dx (\ln x)^2 = x((\ln x)^2 - 2 \ln x + 2) \\
 (4) \int dx x^2 e^x = (x^2 - 2x + 2)e^x & (10) \int dx x (\ln x)^2 = \frac{x^2}{4}(2(\ln x)^2 - 2 \ln x + 1) \\
 (5) \int dx x^2 \cos x = (x^2 - 2) \sin x + 2x \cos x & (11) \int dx e^x \cos x = \frac{e^x}{2}(\cos x + \sin x) \\
 (6) \int dx x^2 \sin x = -(x^2 - 2) \cos x + 2x \sin x & (12) \int dx e^x \sin x = \frac{e^x}{2}(-\cos x + \sin x)
 \end{array}$$

4.

$$\begin{array}{ll}
 (1) \int dx x(x-1)^5 = \frac{1}{7}(x-1)^7 + \frac{1}{6}(x-1)^6 & (7) \int dx \sin^2 x = \frac{1}{4}(2x - \sin(2x)) \\
 (2) \int dx \frac{x}{x+1} = x - \ln|x+1| & (8) \int dx \cos^3 x = \sin x - \frac{1}{3} \sin^3 x \\
 (3) \int \frac{dx}{x^2-1} = \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| & (9) \int dx \sin(2x) \sin(3x) = \frac{1}{10}(5 \sin x - \sin(5x)) \\
 (4) \int dx \frac{x}{\sqrt{x+1}} = \frac{2}{3}(x-2)\sqrt{x+1} & (10) \int dx \sin(x) \cos(2x) = \frac{1}{6}(3 \cos x - \cos(3x)) \\
 (5) \int dx x \sqrt{x-1} = \frac{2}{15}(3x+2)(x-1)^{3/2} & (11) \int \frac{dx}{1+\sin x} = \tan x - \frac{1}{\cos x} \\
 (6) \int \frac{dx}{\sqrt{x^2-1}} = \ln \left| x + \sqrt{x^2-1} \right| & (12) \int \frac{dx}{\sin x} = \frac{1}{2} \ln \left(\frac{1-\cos x}{1+\cos x} \right)
 \end{array}$$

2.

$$\begin{aligned}
 (6) \quad \int dx \sin x \cos x &= \int dx \sin x (\sin x)' = \frac{1}{2} \sin^2 x \\
 \int dx \sin x \cos x &= - \int dx (\cos x)' \cos x = -\frac{1}{2} \cos^2 x \\
 \int dx \sin x \cos x &= \frac{1}{2} \int dx \sin(2x) = -\frac{1}{4} \cos(2x)
 \end{aligned}$$

3.

$$\begin{aligned}
 (11) \quad I &= \int dx e^x \cos x = e^x \cos x + \int dx e^x \sin x = e^x \cos x + e^x \sin x - \int dx e^x \cos x = e^x (\cos x + \sin x) - I \\
 \therefore I &= \int dx e^x \cos x = \frac{e^x}{2} (\cos x + \sin x) \\
 (11), (12) \quad \int dx e^{(1+i)x} &= \frac{1}{1+i} e^{(1+i)x} = \frac{e^x}{2} (1-i) (\cos x + i \sin x) = \frac{e^x}{2} (\cos x + \sin x) + i \cdot \frac{e^x}{2} (-\cos x + \sin x) \\
 \int dx e^{(1+i)x} &= \int dx e^x (\cos x + i \sin x) = \int dx e^x \cos x + i \int dx e^x \sin x
 \end{aligned}$$

4.

$$\begin{aligned}
 (1) \quad \int dx x(x-1)^5 &= \int dx ((x-1)+1)(x-1)^5 = \int dx ((x-1)^6 + (x-1)^5) = \frac{1}{7}(x-1)^7 + \frac{1}{6}(x-1)^6 \\
 (2) \quad \int dx \frac{x}{x+1} &= \int dx \frac{(x+1)-1}{x+1} = \int dx \left(1 - \frac{1}{x+1}\right) = x - \ln|x+1| \\
 (3) \quad \int \frac{dx}{x^2-1} &= \frac{1}{2} \int dx \left(\frac{1}{x-1} - \frac{1}{x+1}\right) = \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| \\
 (4) \quad \int dx \frac{x}{\sqrt{x+1}} &= \int dx \frac{(x+1)-1}{\sqrt{x+1}} = \int dx \left(\sqrt{x+1} - \frac{1}{\sqrt{x+1}}\right) = \frac{2}{3}(x-2)\sqrt{x+1} \\
 (5) \quad \int dx x\sqrt{x-1} &= \int dx ((x-1)+1)\sqrt{x-1} = \int dx \left((x-1)^{3/2} + \sqrt{x-1}\right) = \frac{2}{15}(3x+2)(x-1)^{3/2} \\
 (8) \quad \int dx \cos^3 x &= \int dx (1 - \sin^2 x) \cos x = \sin x - \frac{1}{3} \sin^3 x \\
 \int dx \cos^3 x &= \int dx \frac{1}{4} (3 \cos x + \cos(3x)) = \frac{3}{4} \sin x + \frac{1}{12} \sin(3x) \\
 (10) \quad \int dx \sin(x) \cos(2x) &= \int dx \frac{1}{2} (\sin(3x) - \sin x) = \frac{1}{6} (3 \cos x - \cos(3x)) \\
 \int dx \sin(x) \cos(2x) &= \int dx \sin x (2 \cos^2 x - 1) = -\frac{2}{3} \cos^3 x + \cos x \\
 (11) \quad \int \frac{dx}{1+\sin x} &= \int dx \frac{1-\sin x}{\cos^2 x} = \tan x - \frac{1}{\cos x} \\
 \int \frac{dx}{1+\sin x} &= \int dt \frac{2}{1+t^2} \frac{1}{1+\frac{2t}{1+t^2}} = 2 \int \frac{dt}{(1+t)^2} = -\frac{2}{1+t} = -\frac{1}{1+\tan \frac{x}{2}} \quad \left(t = \tan \frac{x}{2}\right) \\
 (12) \quad \int \frac{dx}{\sin x} &= \int dx \frac{\sin x}{1-\cos^2 x} = \frac{1}{2} \int dx \left(\frac{\sin x}{1-\cos x} + \frac{\sin x}{1+\cos x}\right) = \frac{1}{2} \ln \left(\frac{1-\cos x}{1+\cos x}\right) \\
 \int \frac{dx}{\sin x} &= \int dt \frac{2}{1+t^2} \frac{1}{\frac{2t}{1+t^2}} = \int \frac{dt}{t} = \ln|t| = \ln \left| \tan \frac{x}{2} \right| \quad \left(t = \tan \frac{x}{2}\right)
 \end{aligned}$$