演習：微分の計算【微分積分】

1.

(1) $y = x^{\alpha}$

(2) $y = \dfrac{1}{x}$

(3) $y = \sqrt{x}$

(4) $y = \dfrac{1}{\sqrt{x}}$

(5) $y = \sin x$

(6) $y = \cos x$

(7) $y = \tan x$

(8) $y = \dfrac{1}{\tan x}$

(9) $y = e^{x}$

(10) $y = \ln x$

(11) $y = a^{x}$

(12) $y = \log_{a} x$

2.

(1) $y = (x^2+1)^3$

(2) $y = \dfrac{1}{x^2+1}$

(3) $y = \sqrt{x^2+1}$

(4) $y = \dfrac{1}{\sqrt{x^2+1}}$

(5) $y = (x^2+1)^{3/2}$

(6) $y = x(x^2+1)^2$

(7) $y = x^2(x^2+1)^3$

(8) $y = \dfrac{x}{x^2+1}$

(9) $y = x\sqrt{x^2+1}$

(10) $y = \dfrac{x}{\sqrt{x^2+1}}$

(11) $y = \dfrac{\sqrt{x^2+1}}{x}$

(12) $y = \dfrac{1}{x\sqrt{x^2+1}}$

3.

(1) $y = \sin(x^2)$

(2) $y = \cos\left(\dfrac{1}{x}\right)$

(3) $y = \sin\left(\sqrt{x}\right)$

(4) $y = \cos\left(\cos x\right)$

(5) $y = \sin^3 x$

(6) $y = \dfrac{1}{\sin x}$

(7) $y = \sqrt{\sin x}$

(8) $y = \dfrac{1}{\sqrt{\cos x}}$

(9) $y = x \sin x$

(10) $y = \dfrac{1}{x}\cos\left(\dfrac{1}{x}\right)$

(11) $y = \sin x \cos x$

(12) $y = \sin^2\left(\cos x\right)$

4.

(1) $y = e^{x^2}$

(2) $y = e^{1/x}$

(3) $y = e^{\sin x}$

(4) $y = e^{\sqrt{x}}$

(5) $y = (e^x+1)^3$

(6) $y = \dfrac{1}{e^x+1}$

(7) $y = \sqrt{e^x + e^{-x}}$

(8) $y = \sin(e^x)$

(9) $y = x^2 e^x$

(10) $y = e^x \sin x$

(11) $y = \sin\left(e^{x^2}\right)$

(12) $y = x e^x \cos x$

5.

(1) $y = \ln(x^2+1)$

(2) $y = \ln\left(x + \sqrt{x^2+1}\right)$

(3) $y = \ln(\cos x)$

(4) $y = \ln(e^x + e^{-x})$

(5) $y = (\ln x)^3$

(6) $y = \dfrac{1}{\ln x}$

(7) $y = \sqrt{\ln x}$

(8) $y = \sin(\ln x)$

(9) $y = x \ln x$

(10) $y = \dfrac{1}{x}\ln x$

(11) $y = x^x$

(12) $y = x^{\frac{1}{x}}$

演習：微分の計算【微分積分】解答

1.

(1) $y' = \alpha x^{\alpha-1}$

(2) $y' = -\dfrac{1}{x^2}$

(3) $y' = \dfrac{1}{2\sqrt{x}}$

(4) $y' = -\dfrac{1}{2x\sqrt{x}}$

(5) $y' = \cos x$

(6) $y' = -\sin x$

(7) $y' = \dfrac{1}{\cos^2 x}$

(8) $y' = -\dfrac{1}{\sin^2 x}$

(9) $y' = e^x$

(10) $y' = \dfrac{1}{x}$

(11) $y' = a^x \ln a$

(12) $y' = \dfrac{1}{x \ln a}$

2.

(1) $y' = 6x(x^2+1)^2$

(2) $y' = -\dfrac{2x}{(x^2+1)^2}$

(3) $y' = \dfrac{x}{\sqrt{x^2+1}}$

(4) $y' = -\dfrac{x}{(x^2+1)\sqrt{x^2+1}}$

(5) $y' = 3x(x^2+1)^{1/2}$

(6) $y' = (5x^2+1)(x^2+1)$

(7) $y' = 2x(4x^2+1)(x^2+1)^2$

(8) $y' = \dfrac{1-x^2}{(x^2+1)^2}$

(9) $y' = \dfrac{2x^2+1}{\sqrt{x^2+1}}$

(10) $y' = \dfrac{1}{(x^2+1)\sqrt{x^2+1}}$

(11) $y' = -\dfrac{1}{x^2\sqrt{x^2+1}}$

(12) $y' = -\dfrac{2x^2+1}{x^2(x^2+1)\sqrt{x^2+1}}$

3.

(1) $y' = 2x\cos(x^2)$

(2) $y' = \dfrac{1}{x^2}\sin\left(\dfrac{1}{x}\right)$

(3) $y' = \dfrac{\cos\left(\sqrt{x}\right)}{2\sqrt{x}}$

(4) $y' = \sin x\ \sin(\cos x)$

(5) $y' = 3\sin^2 x\ \cos x$

(6) $y' = -\dfrac{\cos x}{\sin^2 x}$

(7) $y' = \dfrac{\cos x}{2\sqrt{\sin x}}$

(8) $y' = \dfrac{\tan x}{2\sqrt{\cos x}}$

(9) $y' = \sin x + x\cos x$

(10) $y' = -\dfrac{1}{x^2}\cos\left(\dfrac{1}{x}\right) + \dfrac{1}{x^3}\sin\left(\dfrac{1}{x}\right)$

(11) $y' = \cos^2 x - \sin^2 x$

(12) $y' = -2\sin x\ \sin(\cos x)\ \cos(\cos x)$

4.

(1) $y' = 2xe^{x^2}$

(2) $y' = -\dfrac{e^{1/x}}{x^2}$

(3) $y' = e^{\sin x}\cos x$

(4) $y' = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

(5) $y' = 3e^x(e^x+1)^2$

(6) $y' = -\dfrac{e^x}{(e^x+1)^2}$

(7) $y' = \dfrac{e^x - e^{-x}}{2\sqrt{e^x+e^{-x}}}$

(8) $y' = e^x\cos(e^x)$

(9) $y' = x(x+2)e^x$

(10) $y' = e^x(\sin x + \cos x)$

(11) $y' = 2xe^{x^2}\cos\left(e^{x^2}\right)$

(12) $y' = e^x\left((x+1)\cos x - x\sin x\right)$

5.

(1) $y' = \dfrac{2x}{x^2+1}$

(2) $y' = \dfrac{1}{\sqrt{x^2+1}}$

(3) $y' = -\tan x$

(4) $y' = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(5) $y' = \dfrac{3(\ln x)^2}{x}$

(6) $y' = -\dfrac{1}{x(\ln x)^2}$

(7) $y' = \dfrac{1}{2x\sqrt{\ln x}}$

(8) $y' = \dfrac{\cos(\ln x)}{x}$

(9) $y' = \ln x + 1$

(10) $y' = -\dfrac{\ln x - 1}{x^2}$

(11) $y' = x^x(\ln x + 1)$

(12) $y' = -x^{\frac{1}{x}-2}(\ln x - 1)$

演習：微分の計算【微分積分】詳解・別解

1.

(11) $y' = \left(a^x\right)' = \left(e^{x\ln a}\right)' = e^{x\ln a}\ln a = a^x \ln a$

$\dfrac{y'}{y} = (\ln y)' = (x\ln a)' = \ln a \quad \therefore y' = y\ln a = a^x \ln a$

(12) $y' = \left(\log_a x\right)' = \left(\dfrac{\ln x}{\ln a}\right)' = \dfrac{1}{x\ln a}$

2.

(4) $y' = \left(\dfrac{1}{\sqrt{x^2+1}}\right)' = -x(x^2+1)^{-3/2} = -\dfrac{x}{(x^2+1)\sqrt{x^2+1}}$

(6) $y' = \left(x(x^2+1)^2\right)' = (x^2+1)^2 + 4x(x^2+1) = (5x^2+1)(x^2+1)$

(7) $y' = \left(x^2(x^2+1)^3\right)' = 2x(x^2+1)^3 + 6x^3(x^2+1)^2 = 2x(4x^2+1)(x^2+1)^2$

(8) $y' = \left(\dfrac{x}{x^2+1}\right)' = \dfrac{1}{x^2+1} - \dfrac{2x^2}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}$

(9) $y' = \left(x\sqrt{x^2+1}\right)' = (x^2+1)^{1/2} + x^2(x^2+1)^{-1/2} = \dfrac{2x^2+1}{\sqrt{x^2+1}}$

(10) $y' = \left(\dfrac{x}{\sqrt{x^2+1}}\right)' = (x^2+1)^{-1/2} - x^2(x^2+1)^{-3/2} = \dfrac{1}{(x^2+1)\sqrt{x^2+1}}$

(11) $y' = \left(\dfrac{\sqrt{x^2+1}}{x}\right)' = (x^2+1)^{-1/2} - \dfrac{(x^2+1)^{1/2}}{x^2} = -\dfrac{1}{x^2\sqrt{x^2+1}}$

(12) $y' = \left(\dfrac{1}{x\sqrt{x^2+1}}\right)' = -\dfrac{1}{x^2}(x^2+1)^{-1/2} - (x^2+1)^{-3/2} = -\dfrac{2x^2+1}{x^2(x^2+1)\sqrt{x^2+1}}$

3.

(8) $y' = \left(\dfrac{1}{\sqrt{\cos x}}\right)' = \dfrac{1}{2}\sin x(\cos x)^{-3/2} = \dfrac{\tan x}{2\sqrt{\cos x}}$

(11) $y' = (\sin x\cos x)' = \cos^2 x - \sin^2 x = \cos(2x)$

5.

(11) $y' = (x^x)' = (e^{x\ln x})' = e^{x\ln x}(x\ln x)' = x^x(\ln x + 1)$

$\dfrac{y'}{y} = (\ln y)' = (x\ln x)' = \ln x + 1 \quad \therefore y' = y\cdot(\ln x + 1) = x^x(\ln x + 1)$

(12) $y' = \left(x^{\frac{1}{x}}\right)' = (e^{\frac{1}{x}\ln x})' = e^{\frac{1}{x}\ln x}\left(\dfrac{1}{x}\ln x\right)' = -\dfrac{x^{1/x}}{x^2}(\ln x - 1) = -x^{\frac{1}{x}-2}(\ln x - 1)$

$\dfrac{y'}{y} = (\ln y)' = \left(\dfrac{1}{x}\ln x\right)' = -\dfrac{\ln x - 1}{x^2} \quad \therefore y' = y\cdot\left(-\dfrac{\ln x - 1}{x^2}\right) = -\dfrac{x^{1/x}}{x^2}(\ln x - 1) = -x^{\frac{1}{x}-2}(\ln x - 1)$